# Dynamic Backhaul Resource Allocation: An Evolutionary Game Theoretic Approach

Ioannis V. Loumiotis, Evgenia F. Adamopoulou, Konstantinos P. Demestichas, Theodora A. Stamatiadi and Michael E. Theologou

***Abstract*—The recently deployed 4G access technology promises to satisfy the increasing demand of bandwidth consuming applications by providing high network capacity, low latency and seamless mobility. Towards this direction, concept solutions concerning the integration of wireless and optical networks have been proposed. However, the majority of these approaches assume the conventional fixed commitment of resources to the base stations, an inefficient and costly process, especially in case the Passive Optical Network (PON) belongs to a different operator. As a result, new, more dynamic backhaul resource allocation approaches are required. In this paper, the authors study the problem of committing resources of the backhaul network to a base station by employing evolutionary game theory in order to model the interactions between the subscribers and the base station. The asymptotic stability of the proposed scheme is proven under the replicator dynamics model. Finally, the impact of time delay in the proposed scheme is also investigated.**

***Index Terms*—Backhaul resource allocation, converged network, evolutionary game theory, replicator dynamics, delay.**

## I. Introduction

During the last years there has been a tremendous growth of new services in the mobile communication scenery, resulting in an increasing demand for higher data rates so as to guarantee satisfactory quality of experience for the end-users [1]. The advent of 4th Generation (4G) networks promises to address this demand by offering increased capacity, high data rates and seamless mobility [2]. One efficient way of achieving this is through the convergence of the wireless access network with a wired optical network (xPON) in the backhaul [3]. The high data rates that can be offered by the optical backhauling network can achieve the requirements of the 4G standard providing satisfactory Quality of Service (QoS) to the end-users [4].

Proper allocation of the backhaul resources at the base station (BS) is mandatory for the network operator in order to provide an improved QoS experience [5]. Traditionally, network planning has been performed statically and it has been based on empirical methods, which led to a fixed, flat commitment of resources. Though this method yielded satisfactory results in the previous communication standards, it cannot be implemented within a converged optical/wireless network, especially if the backhaul optical network belongs to a different operator. As a result, the necessary resources should be calculated and committed dynamically to each BS. Furthermore, this approach is also motivated by the trend towards self organization [6] in communication networks which introduces the need of BSs with enhanced processing capabilities in contrast to the traditional scheme of a central authority responsible for the functionality of the entire network. Such intelligent BSs will continuously monitor their environment and adapt their behavior when needed, providing robustness against failure.

There are many studies in the literature that deal with the backhaul resource allocation problem. In [7], the authors propose an artificial neural network approach in order to predict the forthcoming needs at the side of the BS, and commit the necessary resources in advance. In [8], Liebl et al. study the fairness problem of backhaul resource allocation for the out-band relay case in LTE-Advanced systems, and compare the performance of different resource allocation strategies in terms of throughput fairness between backhaul and macro access link. In [9], the authors propose a buffer level based backhaul resource partitioning scheme to optimize the resource allocation among relay nodes and macro user equipments in LTE-Advanced. A game theoretic approach for fair bandwidth allocation in Fibre-Wireless (FiWi) access networks has been presented in [10]. The authors proposed an algorithm that relies on the cooperation among relay nodes of the access network, consisting of gateway routers and wireless routers. Specifically, the routers optimize the local traffic (originating from the router) and foreign traffic (forwarded by the router) based on specific sharing and trusting levels among the nodes. However, none of the above approaches takes into consideration the subscriber's dynamic behavior for bandwidth demand, and instead either propose a static solution to the backhaul resource allocation problem or do not consider the effects that the allocation at the wireless section can have on the backhaul network.

In the current paper, the authors elaborate on a game theoretic approach for the backhaul resource allocation problem. In essence, if the subscribers' needs are overestimated, then the commitment of unnecessary resources will reduce the profits of the wireless network operator. On the other hand, if the subscribers' needs are underestimated, then there would be a dissatisfaction of the subscribers and a corresponding cost at the BS. To this end, a novel scheme is proposed that considers the dynamic behavior of the subscribers and achieves proper allocation of the backhaul resources at the BS. Evolutionary game theory is employed in order to model the interactions of

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a BS and its subscribers in the cell. Furthermore, the stability of the proposed scheme is investigated under both replicator dynamics and delayed replicator dynamics. According to the authors' best knowledge, the evolutionary game theoretic approach of the backhaul resource allocation problem has not yet been studied in the relevant literature.

The rest of the paper is organized as follows. In Section II, a brief introduction of evolutionary game theory is presented. Section III formulates the problem and states the relevant assumptions. The implementation of the evolutionary game theoretic framework in the backhaul resource allocation problem is studied in Section IV. In Section V, the impact of time delay to the proposed model is investigated. Simulation results of the proposed scheme are presented in Section VI. Finally, Section VII concludes the paper.

## II. Evolutionary Game Theory Preliminaries

Classical game theory is used widely to model interactions between players. One basic assumption used in game theory is that players are rational and try to maximize their payoffs by taking into account the other players' behavior [11]. As a result, the solution of the game that is formed is a stationary solution. On the other hand, evolutionary game theory does not rely on this rationality assumption. Interacting populations instead of players are considered, and individuals (agents) within a population have fixed strategies and interact randomly with other individuals [12]. The populations can learn, adapt and evolve. In evolutionary game theory, the payoff is interpreted as fitness, and success in the game is interpreted as reproductive success. Strategies that do well (better than average) reproduce faster, and strategies that do poorly (worse than average) are outcompeted [12].

### A. Replicator Dynamics

Replicator dynamics, first proposed by Taylor and Jonker [13], are used to model the game interactions of irrational populations. They describe the evolution of the frequencies of strategies in a population, and can be modeled as a system of ordinary differential equations on the simplex $S_n$ [12]. It is assumed that the frequency $x_i$ of a strategy $i$ is the proportion of the population that uses that strategy. The fitness $\pi_i$ of a pure strategy $i$ is a function of the composition of the population, i.e. of the state $\mathbf{x}$. If the population is very large and the generations blend continuously, it can be assumed that $\mathbf{x}(t)$ evolves in $S_n$ as a differentiable function of $t$. The growth rate $\dot{x_i}/x_i$ of the population share using pure strategy $i$ is a measure of its evolution success [14]. This success can be expressed as the difference between the fitness $\pi_i$ of pure strategy $i$ and the average fitness $\bar{\pi}(x) = \sum_i (x_i \pi_i(\mathbf{x}))$, i.e.

$$\frac{\dot{x_i}}{x_i} = \text{fitness of pure strategy } i - \text{average fitness} \tag{1}$$

which yields the replicator equation

$$\dot{x_i} = x_i \left[\pi_i(\mathbf{x}) - \bar{\pi}(\mathbf{x})\right] \tag{2}$$

for $i = 1, \ldots, n$.

The sum $S = x_1 + \ldots + x_n$ satisfies

$$\dot{S} = (1 - S)\bar{\pi} \tag{3}$$

which has $S(t) = 1$ as a solution. Thus, if the solution of the replicator dynamics starts on the plane $\sum_n x_i = 1$, it remains there [14].

In the case of linear $\pi_i$ a $n \times n$ matrix $\mathbf{A} = (A_{ij})$ exists such that $\pi_i(\mathbf{x}) = (\mathbf{Ax})_i$. So, the equation of the replicator dynamics takes the form [14]

$$\dot{x_i} = x_i \left[(\mathbf{Ax})_i - \mathbf{x} \cdot \mathbf{Ax}\right] \tag{4}$$

### B. Asymmetric Games

A particularly interesting case is that of asymmetric conflicts, i.e. the case in which the players do not have the same set of strategies and payoffs. It is assumed that player I has $n$ strategies and a payoff matrix $\mathbf{A}$, whereas player II has $m$ strategies and a payoff matrix $\mathbf{B}^T$. Thus, player I using strategy $i$ against player II using strategy $j$ obtains the payoff $A_{ij}$ and the opponent obtains $B_{ij}$. The mixed strategy of player I is denoted by $\mathbf{x} \in S_n$ and for player II by $\mathbf{y} \in S_m$. The corresponding payoffs are given by $\mathbf{x} \cdot \mathbf{Ay}$ and $\mathbf{y} \cdot \mathbf{B}^T\mathbf{x}$, respectively [14].

The pair $(\mathbf{x}^*, \mathbf{y}^*) \in S_n \times S_m$ is said to be Nash equilibrium if $\mathbf{x}^*$ is a best reply to $\mathbf{y}^*$ and $\mathbf{y}^*$ a best reply to $\mathbf{x}^*$, i.e. if

$$\mathbf{x} \cdot \mathbf{Ay}^* \leq \mathbf{x}^* \cdot \mathbf{Ay}^* \tag{5}$$

for all $\mathbf{x} \in S_n$ and

$$\mathbf{y} \cdot \mathbf{B}^T\mathbf{x}^* \leq \mathbf{y}^* \cdot \mathbf{B}^T\mathbf{x}^* \tag{6}$$

for all $\mathbf{y} \in S_m$. The set of equilibria for bimatrix games is always nonempty.

Assuming $\mathbf{x} \in S_n$ and $\mathbf{y} \in S_m$ denote the frequencies of the strategies for the players, and fitness $\pi_i$ is linear, then the replicator dynamics take the following form [14]

$$\dot{x_i} = x_i \left[(\mathbf{Ay})_i - \mathbf{x} \cdot \mathbf{Ay}\right] \tag{7a}$$

$$\dot{y_j} = y_j \left[(\mathbf{B}^T\mathbf{x})_j - \mathbf{y} \cdot \mathbf{B}^T\mathbf{x}\right] \tag{7b}$$

for $i = 1, \ldots, n$ and $j = 1, \ldots, m$.

In asymmetric games, it holds that an evolutionary stable strategy (ESS)[1] is an asymptotically stable equilibrium under the replicator dynamics [15]. However, an ESS of an asymmetric evolutionary game must be a strict Nash equilibrium. It is reminded that a strict Nash equilibrium must consist of pure strategies, i.e. players I and II must be monomorphic in equilibrium. As a result, the following theorem holds.

*Theorem 2.1:* A strictly mixed-strategy Nash equilibrium of asymmetric evolutionary games is not an asymptotically stable equilibrium under the replicator dynamics [15].

[1]ESS is a strategy that, if adopted by a population of agents, then no other strategy can invade [12].

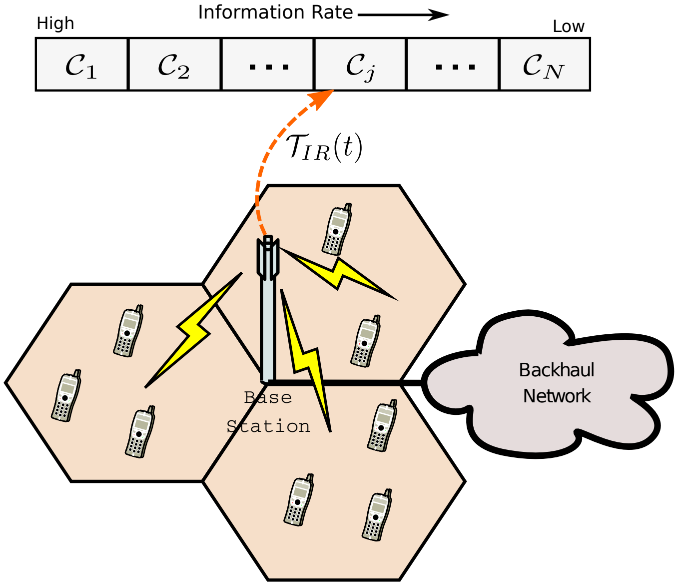


Fig. 1. Network Topology.

## III. Problem Formulation and Relevant Assumptions

Assume that there is a BS and a set $\mathbf{M}$ of subscribers connected to it, as depicted in Fig.(1). If each subscriber at time $t$, requests an information rate (IR) $d_i(t)$, then the aggregated IR requested at the BS at each time $t$, is given by

$$\mathcal{T}_{IR}(t) = \sum_{i \in \mathbf{M}} d_i(t) \tag{8}$$

Hence, the aim of the BS is to correctly commit the necessary IR, $\mathcal{T}_{IR}(t)$, from the backhaul network in order to satisfy the requested demand by the subscribers at time $t$.

Assume for simplicity that the available IR is divided in $N$ classes $\mathcal{C}_i$, $i \in [1, \ldots, N]$, arranged in descending order of IR, so that class $\mathcal{C}_1$ denotes the highest possible IR that can be committed or requested, as can be seen in Fig.(1). If the BS has committed class $\mathcal{C}_i$[2] and subscribers have requested for IR that belongs to class $\mathcal{C}_j$, i.e. $\mathcal{T}_{IR}(t) \in \mathcal{C}_j$, then there are two interesting cases:

- If $i \leq j$, i.e. the committed class $\mathcal{C}_i$ is higher than the requested class $\mathcal{C}_j$, then the BS can satisfy the requested resources. It is assumed that in this case the subscribers experience a payoff $\mathcal{S}_u^{(j)}$ which is a monotonically increasing function of the IR, and the BS experiences a payoff $\mathcal{U}_{BS}^{(i,j)}$ that can be described as

$$\mathcal{U}_{BS}^{(i,j)} = U_{BS}^{(i)} - C_{BS}^{(j-i)} \tag{9}$$

  where $U_{BS}^{(i)}$ is a monotonically increasing function with respect to the IR, so that the BS experiences a higher payoff when more resources are committed, due to the satisfaction of the subscribers and the corresponding revenues. The second term is a penalty function when unnecessary resources are committed, and depends on the service level agreement (SLA) between the wireless network service provider and the PON provider. Specifically, for $i \neq j$, $\mathcal{U}_{BS}^{(i,j)}$ represents the net payoff in the case of overprovisioning, where the BS commits a higher class than requested with a corresponding cost $C_{BS}^{(j-i)}$. It holds that $C_{BS}^{(j-i)} = 0$, for $i = j$.
- If $i > j$, i.e. the committed class $\mathcal{C}_i$ is lower than the requested class $\mathcal{C}_j$, then the BS has committed less resources than requested. In this case, it is expected that only a percentage of the subscribers are satisfied. Hence, the payoff when the subscribers are not fully satisfied by the committed class depends on the payoff $\mathcal{S}_u^{(j)}$ of the requested class and on the difference between the committed and the requested classes. This can be expressed as

$$\mathcal{S}_d^{(i,j)} = \mathcal{S}_u^{(j)} \cdot \left(1 - \frac{i-j}{N}\right) \tag{10}$$

  Similarly, the payoff of the BS depends on the payoff of the committed class discounted by a factor that represents the dissatisfaction of the subscribers and the corresponding cost experienced by the provider due to violation of the end-users' SLAs. This can be expressed as

$$\mathcal{U}_d^{(i,j)} = U_{BS}^{(i)} \cdot (1-\delta)^{(i-j)} \tag{11}$$

  where $0 \leq \delta \leq 1$ depends on the profile of the BS. Specifically, $\delta$ can be considered as the weight that the BS gives to its reputation. For the case of $\delta = 0$, the BS does not care about its inefficiency to satisfy the subscribers' requested class, while for the case of $\delta = 1$, the BS experiences a payoff of zero when the subscribers needs are not satisfied. In particular, payoff $\mathcal{U}_d^{(i,j)}$ exhibits, an exponential decay which prevents the BS from committing less resources and creating unsatisfied demand. In a practical scenario, the choice of $\delta$ may depend on the location of the BS. Specifically, a BS located in an urban area should have strict restrictions concerning the dissatisfaction of the subscribers, which corresponds to a high value of $\delta$. On the other hand, a BS located in a rural area may be more tolerant to its inefficiency in satisfying the subscribers' demand, which in turn corresponds to a low value of $\delta$.

Assuming that $\mathbf{A}$ denotes the payoff matrix of the BS and $\mathbf{B}$ denotes the payoff matrix of the subscribers, then according to the previous analysis it holds that

$$\mathbf{A} = \begin{bmatrix} \mathcal{U}_{BS}^{(1,1)} & \mathcal{U}_{BS}^{(1,2)} & \cdots & \mathcal{U}_{BS}^{(1,N)} \\ \mathcal{U}_{d}^{(2,1)} & \mathcal{U}_{BS}^{(2,2)} & \cdots & \mathcal{U}_{BS}^{(2,N)} \\ \vdots & \vdots & \vdots & \vdots \\ \mathcal{U}_{d}^{(N,1)} & \mathcal{U}_{d}^{(N,2)} & \cdots & \mathcal{U}_{BS}^{(N,N)} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \mathcal{S}_{u}^{(1)} & \mathcal{S}_{u}^{(2)} & \cdots & \mathcal{S}_{u}^{(N)} \\ \mathcal{S}_{d}^{(2,1)} & \mathcal{S}_{u}^{(2)} & \cdots & \mathcal{S}_{u}^{(N)} \\ \vdots & \vdots & \vdots & \vdots \\ \mathcal{S}_{d}^{(N,1)} & \mathcal{S}_{d}^{(N,2)} & \cdots & \mathcal{S}_{u}^{(N)} \end{bmatrix}$$

[2]It refers to the committed IR (CIR), as specified in [16]-[18].

where $A_{ij}$ is the payoff of the BS when class $\mathcal{C}_i$ is committed and class $\mathcal{C}_j$ is requested. Similarly, $B_{ij}$ is the payoff of the subscribers when $\mathcal{C}_j$ is requested, while the BS has committed class $\mathcal{C}_i$.

As can be easily seen from the above analysis, there are two parameters that affect the payoff of the BS. The first is the reputation weight of the BS in Eq.(11), and the second is the cost of unnecessary resources described in Eq.(9). The influence of the former parameter is trivial and can be easily calculated through the above analysis. Concerning the second parameter, it can be assumed that if the cost of unnecessary resources was negligible then the BS would only commit the higher class which corresponds to the worst case scenario. However, because of the high capacity requirements of the 4G networking, planning the network according to the worst case scenario is inefficient. Thereafter, in the following sections, it is considered that the cost of unnecessary resources prohibits the commitment of a class higher than the requested. Thus, the payoff of the BS is higher when it has correctly committed the necessary resources, i.e. $\mathcal{U}_{BS}^{(i,j)} < \mathcal{U}_{BS}^{(j,j)}$ for $i < j \in [1, \ldots, N]$.

## IV. Evolutionary Backhaul Resource Allocation

### A. Evolutionary game dynamics model

The general theoretic framework outlined in Section II is now applied to the backhaul resource allocation problem under consideration. Let $x_i$ and $y_i$ denote the frequency of the population that uses class $\mathcal{C}_i$, $i \in [1, \ldots, N]$, for the BS and the subscribers, respectively. It holds that $\sum_i x_i = 1$ and $\sum_i y_i = 1$. If $\varphi_i$ and $\vartheta_j$ denote the payoff of the BS and subscribers for using class $\mathcal{C}_i$ and $\mathcal{C}_j$, respectively, then it follows that

$$\varphi_i = \sum_{j=1}^{N} A_{ij} y_j \tag{12}$$

$$\vartheta_j = \sum_{i=1}^{N} B_{ij} x_i \tag{13}$$

In the game under consideration, it is assumed that in each period (generation) the BS reconsiders its strategies with probability $r_{BS}$, and only these strategies that yield a higher payoff than average are favored. On the other hand, same actions are taken by the subscribers who reconsider their strategies with probability $r_S$. For a small period of time, the rate of strategy change can be described by the replicator dynamics equations expressed as

$$\dot{x_i} = r_{BS} \cdot x_i(\varphi_i - \bar{\varphi}) \tag{14a}$$

$$\dot{y_j} = r_S \cdot y_j(\vartheta_j - \bar{\vartheta}) \tag{14b}$$

for $i, j = 1, \ldots, N$, where $\bar{\varphi} = \sum_i x_i \varphi_i$ and $\bar{\vartheta} = \sum_i y_i \vartheta_i$ are the average payoffs of the BS and the subscribers, respectively. It can be proved that the following theorem holds.

*Theorem 4.1:* The resources that the BS commits and the resources that the subscribers request will converge.

*Proof:* The proof is given in Appendix A. ∎

This theorem asserts that the game will be settled to a rest point that corresponds to a pure strategy Nash Equilibrium and will not oscillate chaotically. Hence, the interactions between the BS and the subscribers will lead the system to a state that they both commit and request the same amount of resources. Specifically, after a certain amount of interacting periods and depending on the initial state, the subscribers will learn the availability of class $\mathcal{C}_i$ and will modify their request in order to increase their payoff. At the same time, the BS will learn the class $\mathcal{C}_j$ that is preferred by the subscribers and adapt its strategy dynamically in order to increase its own payoff. Finally, the requested class and the committed class will converge and the game will be settled to its equilibrium point.

In the above analysis, it has been assumed that there is a cost incurred to the BS when unnecessary resources are committed. This corresponds to the case that the wireless access network and the backhaul optical network belong to different operators. Assuming that the same operator owns both the access and the backhaul network and cannot lease the unused resources of the backhaul network to another operator, it follows that the above cost is zero. It is evident that in this case, the strategy of the BS to commit class $\mathcal{C}_1$ becomes a strictly dominant strategy and as a result it is an ESS for the BS. Hence, according to Theorem 4.1, the requested class by the subscribers will converge to class $\mathcal{C}_1$.

### B. Discussion

In the previous subsection, it was proven that the committed and the requested resources will converge under the replicator dynamics. In order to illustrate the interpretation of this result in a real implementation scenario, assume that the BS commits resources[3] belonging to class $\mathcal{C}_i$ while the subscribers' aggregated demand belongs to class $\mathcal{C}_j$. It is expected that when the committed class is lower than the class requested by the subscribers, the latter will experience a degraded QoS provisioning, which for instance may correspond to a low video quality, a low downlink speed, etc., depending on their requirements. Thereafter, the BS should commit more resources in order to satisfy the high demand. At the same time, the subscribers experience a degraded QoS provisioning which increases their dissatisfaction[4] and results in the reduction of the aggregated demand. This reduction corresponds to different scenarios such as the subscribers being "forced" to lower their requests, as they cannot be served, or switch to a different available access technology (e.g. Wi-Fi). Specifically, in the former scenario, the subscribers can either simply tolerate a degraded QoS experience, which corresponds to low values of certain quality parameters like video resolution and audio bit rate, or even cease using a requested service (e.g. in case of very low quality video streaming). Consequently, both the BS and the subscribers will adapt their behavior leading the system to an equilibrium point.

Similarly, if the committed resources exceed the resources requested by the subscribers, then the BS should decommit a percentage of them in order to increase its monetary income.

[3] The committed IR is given in discrete steps as proposed in [18].

[4] Despite the subscribers' dissatisfaction, it is considered unlikely for them to change their current service provider due to the contractual nature of customer-operator relationships [19].

However, as the subscribers experience a high QoS, they increase their aggregated demand by requesting more services at an individual level (e.g. file downloading and browsing or watching video streaming). Accordingly, in this case the BS and the subscribers adapt their behavior leading the system to an equilibrium point in which the committed and the requested resources coincide.

The adaptive behavior described above experiences significant differences between the BS and the subscribers, which are reflected by the values of the reviewing probabilities $r_{BS}$ and $r_S$ introduced in the previous subsection. Specifically, it is expected that the subscribers would present a more adaptive behavior than the BS, as the latter is limited by the latencies and the technical restrictions of the network elements.

## V. Time Delay model

In the previous section the asymptotic stability of the replicator dynamics model was proven. However, one of the major drawbacks of the replicator dynamics model is the assumption that the players know the other players' strategies immediately. So, the players decide their strategies based on this knowledge. In this section, the authors study the more realistic case of the decision under outdated information. It is assumed that the BS is informed about the choices of the subscribers with a delay of $\tau_1$. Accordingly, the subscribers learn the strategy of the BS with a delay of $\tau_2$. As a result, the payoff of strategy $i$ for the BS is the payoff $\tau_1$ time units ago, and for the subscribers is $\tau_2$ time units ago. The system takes the form

$$\dot{x_i} = r_{BS} \cdot x_i(\varphi_i^{\tau_1} - \bar{\varphi}^{\tau_1}) \tag{15a}$$
$$\dot{y_j} = r_S \cdot y_j(\vartheta_j^{\tau_2} - \bar{\vartheta}^{\tau_2}) \tag{15b}$$

for $i, j = 1, \ldots, N$, where $\varphi_i^{\tau_1} = \sum_{j=1}^{N} A_{ij} y_j(t-\tau_1)$ is the delayed payoff experienced by the BS for using class $\mathcal{C}_i$, $\bar{\varphi}^{\tau_1} = \sum_i x_i \varphi_i^{\tau_1}$ is its average payoff, $\vartheta_j^{\tau_2} = \sum_{i=1}^{N} B_{ij} x_i(t-\tau_2)$ is the delayed payoff experienced by the subscribers for requesting class $\mathcal{C}_j$ and $\bar{\vartheta}^{\tau_2} = \sum_i y_i \vartheta_i^{\tau_2}$ is their average payoff. The following theorem holds.

*Theorem 5.1:* Under the delays of $\tau_1$ and $\tau_2$, the resources that the BS commits and the resources that the subscribers request will converge, in the long run.

*Proof:* The proof is given in Appendix B. ■

The result of Theorem 5.1 is of great importance as it proves that the game will be settled to an equilibrium state in which both the subscribers and the BS request and commit the same class, even under the impact of a delay to the knowledge of the players.

## VI. Simulation Results

In this section, the authors validate the proposed scheme via simulation results. To this end, payoff functions consistent with the analysis presented in Section III are selected and the profiles of the players are formed. Specifically, it is assumed that the payoff $\mathcal{S}_u^{(j)}$ of the subscribers is described by the sigmoid function

$$\mathcal{S}_u^{(j)} = \frac{\mu}{1 + \nu \cdot exp\left[-(N-j)\xi + \sigma\right]} \tag{16}$$

| Base Station | | Subscribers | |
|---|---|---|---|
| Parameter | Value | Parameter | Value |
| $\epsilon$ | 4 | $\mu$ | 8 |
| $\zeta$ | 0.1 | $\nu$ | 1 |
| $\omega$ | 1.2 | $\xi$ | 0.1 |
| $\delta$ | 0.4 | $\sigma$ | 1 |

TABLE I
Parameter values for the profiles of the BS and the subscribers.

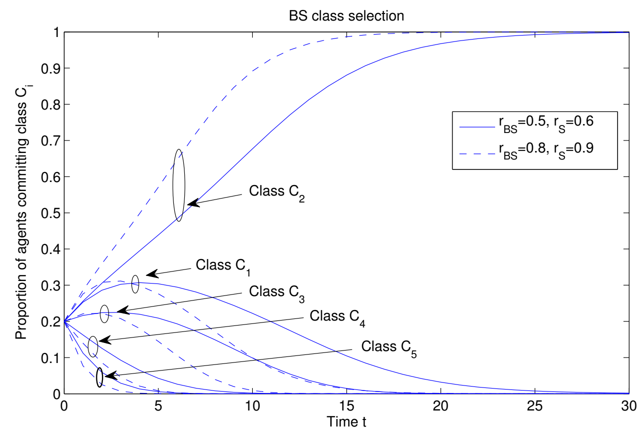


Fig. 2. Evolution of the committed classes of the BS. The parameter values of the players' profiles are given in Table I.

where $\nu, \mu, \xi, \sigma > 0$ are parameters dependent on the profile of the subscribers. Furthermore, the payoff $U_{BS}^{(i)}$ for the BS is given by the exponential decay function with respect to the committed class $\mathcal{C}_i$

$$U_{BS}^{(i)} = \epsilon \cdot e^{(N-i)\zeta} \tag{17}$$

where $\epsilon, \zeta > 0$ are parameters dependent on the profile of the BS. Similar functions have been used in [20] to depict the satisfaction of both the end-users and the network provider in the radio access technology (RAT) selection optimization problem in heterogeneous wireless networks.

Finally, the cost of unnecessary resources is assumed to be linear with respect to the difference between requested and committed class, i.e.

$$C_{BS}^{(j-i)} = \omega \cdot (j - i) \tag{18}$$

where $\omega > 0$ are parameters dependent on the profile of the BS. The parameter values of the profiles of the players are given in Table I.

Consider the case of $N = 5$ classes. In Fig.(2) and in Fig.(3) the evolution of the strategies of the BS and the subscribers are depicted, respectively. It is obvious that there is a fast convergence of the committed class by the BS and the requested class by the subscribers. Although the convergence of the strategies of the subscribers and the BS is assured, the class $C_i$ in which both strategies will converge depends on the initial conditions and the parameter values of the profiles of the players which define the basins of attraction of the equilibrium points. However, this analysis is beyond the scope

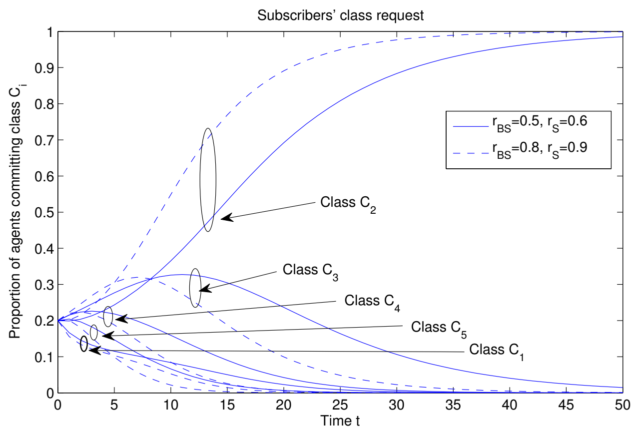


Fig. 3. Evolution of the requested classes of the subscribers. The parameter values of the players' profiles are given in Table I.

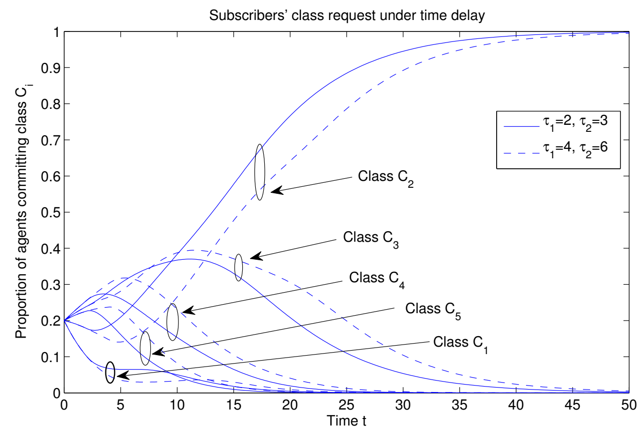


Fig. 5. Evolution of the requested classes of the subscribers for $r_{BS} = 0.8$ and $r_S = 0.9$. The parameter values of the players' profiles are given in Table I.

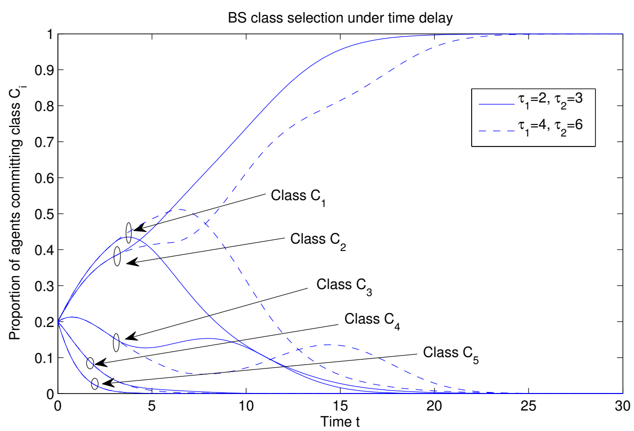


Fig. 4. Evolution of the committed classes of the BS for $r_{BS} = 0.8$ and $r_S = 0.9$. The parameter values of the players' profiles are given in Table I.

of the current paper and the reader is referred to [21]-[23] for more details.

Furthermore, the influence of the reviewing probabilities $r_{BS}$ and $r_S$ on the convergence time is also illustrated in Fig.(2) and in Fig.(3). It is evident that when the BS and the subscribers review their strategies more often, the convergence speed to the equilibrium is increased.

The impact of time delay on the strategic choices of the BS and the subscribers is depicted in Fig.(4) and in Fig.(5), respectively. Although both players' strategies converge to the same class as it was proven in Appendix B, it becomes apparent that there is a negative impact of time delay to the convergence speed. However, Theorem 5.1 asserts that even for high values of time delays $\tau_1$ and $\tau_2$, the system will converge to an equilibrium state. It should be noted that the equilibrium state under the classical replicator dynamics and the delayed replicator dynamics is not necessarily the same, even for the same initial conditions.

## VII. Conclusion and Future Research

In the current paper, the authors studied the problem of backhaul resource allocation at the side of the BS. Evolutionary game theory was employed to model the interactions of a BS and the population of subscribers. It was shown that, under the replicator dynamics, the commitment of the resources and the subscribers' demand of IR will converge. In this way, the BS will commit only the necessary IR to the subscribers, achieving a better allocation of the backhaul resources. Finally, it was proven that the proposed scheme retains its asymptotic stability nature under time delay.

In general, the backhaul resource allocation problem embraces a vast research area in a converged network architecture. As stated above, the current approach elaborates on this problem from the perspective of the BS, considering that appropriate SLAs between the PON operator and the wireless network service provider should ensure both parties in providing QoS to their customers. Nevertheless, the clients of the PON may include other wireless network service providers or corporate users who compete among themselves and bid for its resources. Thus, there is a different perspective of the backhaul resource allocation problem encountered from the side of the PON, which is not studied in the current work. Furthermore, in the proposed scheme, the concept of the aggregated demand was used to represent the requirements of the subscribers. An even more finegrained approach would require from the subscribers to be clustered in groups sharing common needs, selecting among different class of services and interacting with the BS as distinct groups. Both of the above approaches could extend the currently proposed scheme and constitute directions for future research.

## Appendix A
## Proof of Theorem 4.1

It is trivial to show that there exist $N$ pure strategy Nash Equilibrium (NE), namely $(\mathcal{C}_k, \mathcal{C}_k)$ for $k \in [1, \ldots, N]$ and one mixed strategy NE. According to Theorem 2.1, the latter cannot be asymptotically stable, and, thus, it is not investigated. For the former equilibrium points, using linearization techniques, the $k$-th Jacobian matrix calculated at the equilibrium point $(\mathcal{C}_k, \mathcal{C}_k)$, $k \in [1, \ldots, N]$, can be expressed in block matrices as

$$\mathbf{J}^{(k)} = \left[ \begin{array}{c|c} \mathbf{W} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{Q} \end{array} \right]$$

where $\mathbf{W}$ and $\mathbf{Q}$ are $(N-1) \times (N-1)$ matrices with elements

$$W_{ii} = \begin{cases} r_{BS}\left(A_{ik} - A_{kk}\right) & \text{if } i \neq k \\ r_{BS}\left(A_{Nk} - A_{kk}\right) & \text{if } i = k \end{cases} \tag{19a}$$

$$W_{ij} = \begin{cases} 0 & \text{if } i \neq k \\ r_{BS}\left(A_{Nk} - A_{jk}\right) & \text{if } i = k \end{cases} \tag{19b}$$

$$Q_{ii} = \begin{cases} r_{S}\left(B_{ki} - B_{kk}\right) & \text{if } i \neq k \\ r_{S}\left(B_{kN} - B_{kk}\right) & \text{if } i = k \end{cases} \tag{19c}$$

$$Q_{ij} = \begin{cases} 0 & \text{if } i \neq k \\ r_{S}\left(B_{kN} - B_{kj}\right) & \text{if } i = k \end{cases} \tag{19d}$$

Using simple block matrices algebra [24], it can be found that the eigenvalues of the system for the $k$-th equilibrium point, $k \in [1, \ldots, N]$, are described by

$$\lambda_i^{(k)} = \begin{cases} r_{BS}\left(A_{ik} - A_{kk}\right) & \text{if } i \neq k \\ r_{BS}\left(A_{Nk} - A_{kk}\right) & \text{if } i = k \end{cases} \tag{20a}$$

$$\lambda_{N-1+i}^{(k)} = \begin{cases} r_{S}\left(B_{ki} - B_{kk}\right) & \text{if } i \neq k \\ r_{S}\left(B_{kN} - B_{kk}\right) & \text{if } i = k \end{cases} \tag{20b}$$

for $i = 1, \ldots, N-1$. According to the assumptions made in Section III, all the eigenvalues have negative real part. Hence, the $N$ pure strategy NE, $(\mathcal{C}_k, \mathcal{C}_k)$ for $k \in [1, \ldots, N]$, are asymptotically stable and, as a result, the resources that the BS commits and the resources that the subscribers request will converge.

## Appendix B
## Proof of Theorem 5.1

Using linearization techniques, the characteristic polynomial calculated at the $k$-th equilibrium point is given by

$$\mid \mathbf{J}_0^k + e^{-\lambda\tau_1}\mathbf{J}_{\tau_1}^k + e^{-\lambda\tau_2}\mathbf{J}_{\tau_2}^k - \lambda\mathbf{I} \mid = 0 \tag{21}$$

where $\mathbf{J}_0^k$ is the Jacobian of the system calculated with respect to $(\mathbf{x}_k(t), \mathbf{y}_k(t))$, $\mathbf{J}_{\tau_1}^k$ is the Jacobian calculated with respect to $(\mathbf{x}_k(t-\tau_1), \mathbf{y}_k(t-\tau_1))$ and $\mathbf{J}_{\tau_2}^k$ is the Jacobian calculated with respect to $(\mathbf{x}_k(t-\tau_2), \mathbf{y}_k(t-\tau_2))$.

In the same way as in Appendix A, it can be found that at the equilibrium points it holds that $\mathbf{J}_{\tau_1}^k = \mathbf{0}$ and $\mathbf{J}_{\tau_2}^k = \mathbf{0}$. As a result, the stability analysis of the system is the same as in Appendix A.

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